

# **ghyper**, a package for simplification of generalized hypergeometric functions

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This note describes the **ghyper** package of REDUCE, which is able to do simplification of several cases of generalized hypergeometric functions. The simplifications are performed towards polynomials, elementary or special functions or simpler hypergeometric functions. Therefore this package should be used together with the REDUCE special function package.

## **1 Introduction**

The (generalized) hypergeometric functions

$${}_pF_q \left( \begin{matrix} a_1, \dots, a_p \\ b_1, \dots, b_q \end{matrix} \middle| z \right)$$

are defined in textbooks on special functions, e.g. in [1]. Many well-known functions belong to this class, e.g. exponentials, logarithms, trigonometric functions and Bessel functions. In [2] an introduction into the analysis of sums, basic identities and applications can be found.

Several hundreds of particular values can be found in [1].

## 2 REDUCE operator hypergeometric

The operator `hypergeometric` expects 3 arguments, namely the list of upper parameters (which may be empty), the list of lower parameters (which may be empty too), and the argument, e.g:

```
hypergeometric ({} , {} , z);
```

```
  Z
  E
```

```
hypergeometric ({1/2,1} , {3/2} , -x^2);
```

```
  ATAN(X)
  -----
  X
```

## 3 Enlarging the hypergeometric operator

Since hundreds of particular cases for the generalized hypergeometric functions can be found in the literature, one cannot expect that all cases are known to the `hypergeometric` operator. Nevertheless the set of special cases can be augmented by adding rules to the REDUCE system, e.g.

```
let {hypergeometric({1/2,1/2} , {3/2} , -(~x)^2) => asinh(x)/x};
```

## References

- [1] A. P. Prudnikov, Yu. A. Brychkov, O. I. Marichev, *Integrals and Series, Volume 3: More special functions*, Gordon and Breach Science Publishers (1990).
- [2] R. L. Graham, D. E. Knuth, O. Patashnik, *Concrete Mathematics*, Addison-Wesley Publishing Company (1989).